PD Research Report for the 2014 year

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Research Theme Study of Computational Models
Research Period April, 2014 ~ December, 2014
Research Results（about 2,500 characters in Japanese, about 65 lines times 90 characters in English）

The purpose of this research is to formally check the correctness of qualitative spatial reasoning methods (QSR) using proof assistant. The research progress and results for the 2014 year are summarized in the following.

- The formalization in Isabelle/HOL of superposition of rectangles (explained in more details in Research Report for the 2013 year) has been completed and published in [1]. The Isabelle/HOL theories of matrix representation of rectangle superposition are available online [4].

- The above result is limited to 29 rectangular structures. We therefore worked on overcoming this limitation to cover superpositions of any rectangular structures, i.e. generalization of the qualitative spatial method of superposition of rectangles. The generalization requires flexible adjustment of granularity level to establish the direction relations between regions that are not connected. An inductive definition is given to move from one granularity level to a higher one. As a consequence of the generalization:
  - The definition of expressiveness of the qualitative matrix representation is further clarified.
  - The notion of core is re-defined and, therefore, several solutions of superposition can be computed.
  - The properties of success and effectiveness are revised accordingly.

- We are formalizing Allen's interval algebra in Isabelle/HOL and MetiTarski. The temporal relations of Allen's interval algebra are interpreted over the field of reals. Therefore, the interpretation is given as linear multivariate polynomial equalities and inequalities. We used cylindrical algebraic decomposition method provided by the tool QEPCAD in MetiTarski to automatically check the correctness of the composition table of these relations. Although cylindrical algebraic decomposition is of higher complexity, its application to the polynomials generated from the temporal relations terminates in a reasonable time.

- The formalization in Isabelle/HOL is also based on algebraic interpretation over the field of reals. Axioms of Allen's interval algebra have been defined and temporal relations have been formally deduced from them. The proofs of the properties of the axiomatic system is still in progress, though.

List of peer-reviewed publications for the 2014 year

[1] F. Ghourabi and K. Takahashi. Formalizing the Qualitative Superposition of Rectangles in Proof Assistant Isabelle/HOL. In *Proceedings of the 7th International Conference on Agents and Artificial Intelligence (ICAART 2015)*. pages 530 -- 539, Lisbon, Portugal, 2015.

[2] T. Ida, F. Ghourabi, and K. Takahashi. Formalizing Polygonal Knot Origami. *Journal of Symbolic Computation*. doi:10.1016/j.jsc.2014.09.031. 2014. (status: accepted and available online, September 2014)

List of presentations for the 2014 year

[3] F. Ghourabi and K. Takahashi. Formalization of Matrix Representation of Direction Relations with Applications to the Superposition of rectangles. Theorem Proving and Provers metting (TPP 2014, Fukuoka)

Online publications

[4] F. Ghourabi and K. Takahashi. *Isabelle/HOL Theories of the Matrix Representation of Superposition of Rectangles*, November 2014. http://ist.ksc.kwansei.ac.jp/~ktaka/SuperpositionTheory/

List of planned publications

[5] F. Ghourabi, T. Ida, and K. Takahashi, Automated Construction and Proving of Knot Fold by Eos System. In *Origami^6: Post-proceedings of The 6th International Conference on Origami in Science, Mathematics and Education (6OSME2014)*, 2015. (status: will be accepted after minor modifications, January 2015)