Kwansei Gakuin University
Report of Research Outcome

2025/03/14

To President

Department : Science and Technology
Position : Postdoctoral fellow
Name : PHAN THANH HUYEN

I report the outcome of the research as follows.

Name of the Fund/Program	□Sabbatical leave with grant □Sabbatical leave with no grant □KGU Joint Research □Individual Special Research ⊠Postdoctoral fellow ※Please report by designated form as for “International Research Collaboration”.
Research Theme	Theoretical Studies for Topological Design of Photonic Crystals
Research Site/Venue	Kwansei Gakuin University – Graduate School of Science and Technology
Research period	2024/04/01 ～ 2025/03/31 （12 months）

◆ **Summary of the research outcome** （**approx. 2,500 words**）
Please write down the outcomes in detail regarding the research theme above.

Photonic crystals (PhCs), which are the optical analog of the conventional crystals, are attracting more and more attention by reason of their possible applications in telecommunication technology and devices. These photonic materials help controlling the flow of light or electromagnetic (EM) wave. Photons in PhCs can also expose some interesting effects, which are in analogy with electrons in crystals. Although PhCs are very potential materials, the energy-loss due to fabrication and intrinsic properties limit the further development of photonic devices. To overcome this problem, the cross field of “topology”, which is originated from mathematics, and PhCs come into play. This concept is used to distinguish the geometry lattice by some invariant quantities during continuous deformation. The recent studies on topological waveguide states based on PhCs have proven the very low energy-loss properties compared to the normal waveguide, which stimulate the research of topological properties of PhCs.

The purpose of our researches is to theoretically design new PhC structures and find out their topological characteristics by numerical calculations. We also try to optimize the existent photonic structures to make them suitable for each purpose. Along with the study of reducing energy-loss in photonic structures, we will also study the topological properties in non-Hermitian photonic systems.

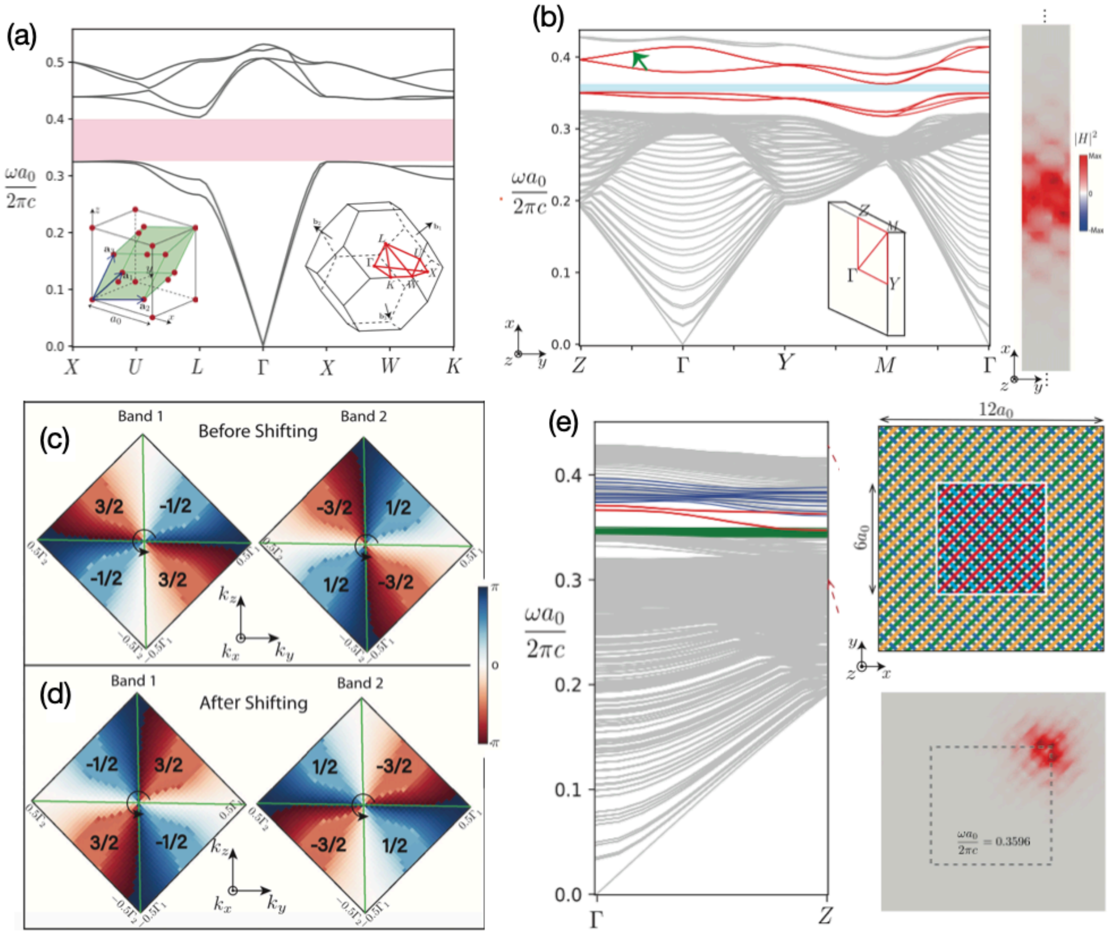

Figure 1: (a) Photonic band structure of the woodpile photonic crystal (calculated from primitive unit cell). The insets show a schematic of a primitive woodpile unit cell and the corresponding first Brillouin zone. This structure shows a relatively large band gap. (b) Photonic dispersion for the interface structure. The red lines denotes interface states, which are in the band gap and isolated from others. A field profile of an interface states labeled by green arrow is shown on the right. (c) and (d) Wilson loop on $k_y k_z$ plane for before- and after-shifting primitive unit cells, respectively. The black bold half-integers indicate winding number of the Wilson loop on each part. (e)Schematic of a supercell containing 4 hinges and its photonic dispersion. The isolated hinge states (red lines) are found at the band gap between interface states. A field profile of a hinge state is shown on the right side.

Following our previous study about microwave hinge states in a 3D simple cubic photonic crystal [1], we have extended our study to the system with reduced symmetry, named woodpile photonic crystal. This structure is derived from the diamond cubic lattice. Its photonic band structure shows a relatively large photonic band gap. To create the difference in topological properties, we introduce the shifting structure, where the origin of the original unit cell is shifted by (¼, ¼, ½) of a lattice constant. The topological interface states emerge at the interface between 2 types of structure are numerically obtained in the complete band gap. A numerical method is introduced to calculate the Wilson loop for woodpile structure. The interface states are topologically protected by the finite difference in the winding number of the Wilson loop. We also examine the hinge states for this woodpile structure and found that the emergence of the hinge states is due to the sign flip of the partial Chern number difference and follows a selection rule.

Most recent studies of topology in three-dimensional structures focus on systems with high symmetry (preserving both inversion and time-reversal symmetry), where topological properties can be definitively predicted from wavefunctions. However, studies of less symmetric 3D systems often describe topological properties in ambiguous terms. Our work offers a robust method for precisely determining topological invariants through calculations, rather than relying on wavefunction-based predictions. We also note that while higher-order topological states have been extensively studied in sonic crystals, research in photonic crystals (PhCs) remains limited. Moreover, our proposed woodpile structure demonstrates the emergence of topological states within a complete band gap, isolated from other states. This design facilitates the confinement of topological states to interfaces or hinges, significantly minimizing energy loss compared to our previously studied simple cubic structure.

Beside the study of 3D woodpile photonic crystal, we also parallelly study the topological properties of 2D non-Hermitian photonic crystal.

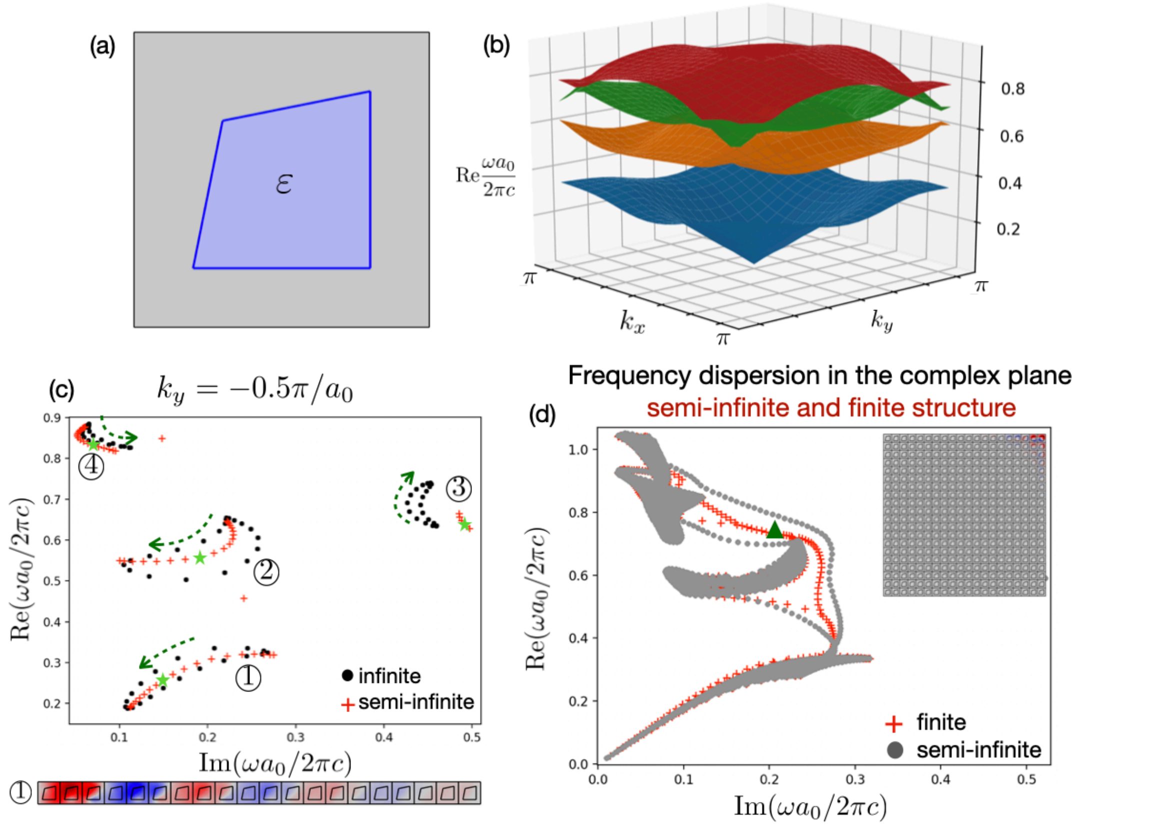

Figure 2: (a) A schematic of a unit cell of one unit cell of a 2D non-Hermitian photonic crystal. The relative susceptibility of dielectric material (blue) is an asymmetric tensor. (b) Photonic band structure of 2D non-Hermitian photonic crystal in (a), where the first band is isolated from other bands. (c) Complex eigenfrequencies of bulk (black) and semi-finite structure (red) in the same complex plane. The inset shows a state, where EM wave is confined to the edge. (d) Complex eigenfrequencies of semi-finite stripe (grey) and finite structure (red) in the same complex plane. The inset shows a corner state labeled by the green triangle.

In this study, we theoretically design a 2D non-Hermitian photonic crystal in broken of both inversion and mirror symmetries. Relative permittivity of the dielectric material is an asymmetric tensor. Therefore, this photonic crystal is nonreciprocal. Photonic dispersion of this structure shows a complete band gap between the first and the second bands. Due to the nonreciprocity, when either k_x or k_y is fixed, these complex eigenfrequencies form closed loop in complex plane. A semi-infinite structure, which is periodic along y-direction and finite in x-direction, is also examined. The eigenfrequencies for this structure inside the closed loop when plotting in the same complex plane. These states exhibit skin effect, where electromagnetic waves are confined at one edge of the structure as shown in Fig. 2(c). Similarly, the eigenfrequencies of semi-infinite structure also form closed loop in complex plane. The second order skin effect, where electromagnetic waves are confined to the corner of finite structure, are also numerically observed as shown in Fig. 2(d). A winding number of each point on complex plane is determined based on whether its position lies inside or outside the closed loop. Eigenfrequencies of states exhibit skin effects are inside the loop. These skin modes are topologically protected by non-zero winding number. This study suggest a way to fine electromagnetic waves in non-Hermitian photonic crystals.

For the future perspectives, we will study the topological phase transition in non-Hermitian photonic systems, examine the propagation/localization of topological modes in non-Hermitian systems. We expect that our studies will impact not only photonic systema, but also other fields of study such as topological insulators, semiconductors and quantum phase transition in general.

References
[1] S. Takahashi, Y. Ashida, **H. T. Phan**, et al., Phys. Rev. B **109** 125304 (2024).

Published papers:
[2] **H.T. Phan**, S. Takahashi, et el., Phys. Rev. B **110** 235429 (2024).

Academic conferences:
[3] **H. T. Phan** and K. Wakabayashi, "Numerical calculation of topological invariants in non-hermitian photonic crystals," in *JSAP Spring Meeting Mar. 14th 2025.*
[4] **H. T. Phan** and K. Wakabayashi, "Topological skin modes in 2d non-Hermitian photonic crystal," in *JST CREST Joint Area Seminar*, Nov. 28th 2024.
[5] **H. T. Phan** and K. Wakabayashi, "Corner skin modes in non-Hermitian photonic crystal," in *JPS Autumn Meeting Sep. 16th 2024.*

Deadline : Within two months after finishing the research period.
Sabbatical leave with grant: Submit this report to President with confirmation by the dean of school you belong to.
※Postdoctoral fellow is required to submit this report with confirmation by the dean of graduate school before the end of employment period.
Where to submit : Organization for Research and Development and Outreach (NUC)
◆We put this report on the web of KGU. If there is any problem about it because of difficulties on your research, please let us know.