PD Research Report for the 2015 year

Name (Research group) BELMONTE Rémy (Katoh Lab) ○

(○ Research Group, Graduate School of Science and Technology)

Research Theme Graph algorithms, parameterized algorithms and combinatorial optimization・ ○

Research Period ○ April, 2015 ~ March, 2016

Research Results（about 2,500 characters in Japanese, about 65 lines times 90 characters in English）

During the academic year 2015, my research focused mainly on three different topics related to the fields of algorithmic graph theory and parameterized algorithms.

The first such topic is related to the fundamental problem of Graph Isomorphism. Given two graphs G and H, the Graph Isomorphism problem asks whether G and H are isomorphic, i.e., whether there is one-to-one mapping from the vertices of G to the vertices of H such that two vertices are adjacent in G if and only if the vertices of H to which they are mapped are adjacent as well. The Graph Isomorphism problem is one of the most fundamental and well-studied problems in algorithmic graph theory as well as pure graph theory. From a classical computational complexity point of view, Graph Isomorphism is notorious for being one of the few problems which are known to lie in the class NP, but it is not known whether the problem can be solved in polynomial time or is NP-complete (we work under the standard assumption that P ≠ NP). It is important to note that, this year, Laslo Babai made a major breakthrough by showing that Graph Isomorphism can be solved in quasi-polynomial time. I worked, together with Dr. Yota Otachi (JAIST, Japan) and Dr. Pascal Schweitzer (RWTH Aachen, Germany), on the classical computational complexity classification of the Graph Isomorphism problem on special classes of graphs. Since the problem is not known to be tractable in the general case, it is common to try and identify classes of graphs for which the problem can be solved efficiently, and those for which it remains as difficult as the general case. Classes for which the problem can be solved in polynomial time include planar graphs, cographs, graphs of bounded treewidth, and more generally graphs that exclude a fixed graph as a minor or a topological minor. On the other note, while the problem is tractable on cographs, which are equivalent to those graphs that do not contain a path on 4 vertices as an induced subgraph, it is known that the class of graphs that does not contain some fixed graph J as an induced subgraph does not admit a polynomial time algorithm for Graph Isomorphism unless J is an induced subgraph of P_4, the induced path on 4 vertices. Seeing as forbidding any fixed graph as a minor yields a tractable case, while forbidding a fixed graph as an induced subgraph generally doesn't help with tractability, it is natural to ask about forbidding a graph in a way stronger than minor, but weaker than induced subgraph. Such a containment relation is the induced minor relation, and we gave a complete dichotomy for the complexity of Graph Isomorphism on induced minor-free graphs. A similar work had been performed by Ponomarenko in 1991, however one of the cases was missing, and one of the proofs was wrong. We solve the missing case and provide a different proof for the other one. Additionally, we simplify and improve another of the three main cases of Ponomarenko's paper. Finally, as a by-product, we provide a dichotomy for the clique-width of induced minor-free graphs, classifying those classes which have bounded clique-width and those that do not. These results were published at the WG 2015 conference, and the journal version is currently under preparation.

The second project on which I worked this year concerns the Metric Dimension problem on graphs of bounded treelength and bounded modular-width. A graph has metric dimension k if there is a set of at most k vertices such that any two vertices of the graph do not lie at the same distance from at least one of those k vertices. The Metric Dimension problem is well-known for having applications in network identification, reliability and fault-tolerance. As part of a collaboration with Pr. Fedor Fomin, Dr. Petr Golovach and Dr. M.S. Ramanoujan from the University of Bergen, Norway, we showed that the Metric Dimension problem can be solved in FPT time when parameterized by the solution size k and the treelength of the input graph, or when parameterized by the modular-width of the input graph alone. Note that it is currently an open problem to determine whether the Metric Dimension problem admits an FPT algorithm when parameterized by the solution size k and the treewidth of the input graph. Treewidth is a well-known graph width measure which represents how tree-like a given graph is. Treelength has a seemingly similar definition, but is known for having a widely different behavior to that of treewidth, and in particular many well-known and well-studied classes of graphs have bounded treelength, such as chordal graphs and AT-free graphs. Our results imply in particular that Metric Dimension is FPT on chordal graphs and AT-free graphs when parameterized by the solution size. This answers a recent open question of Foucaud et al., who showed that the problem is FPT on interval graphs, a proper subclass of chordal graphs. These results were published at the MFCS 2015 conference, and the journal version was submitted for publication.

The third problem on which I have worked this year is a collaboration with Dr. Yota Otachi from JAIST, Japan. In this work, we initiate the study of a problem which we refer to as Generalized Distance Domination. Domination problems is a large and well-studied family of problems in algorithmic graph theory which includes fundamental problems such as Minimum Dominating Set and Maximum Independent Set. In the classical Generalized Domination problem, one must find a set of vertices of the input graph that satisfies constraints related to the number of neighbors each vertex inside and outside of the solution set has inside and outside of the solution set. The variant we introduce imposes constraints on the distance between vertices and the solution set instead of the number of neighbors. Our main result is to show that the problem can be solved in polynomial time on the class of AT-free graphs and circular permutation graphs. This complements similar results for the classical Generalized Domination problem. These results are currently under preparation for submission to a conference.

I have also performed various research-related academic duties, including a dozen reviews for various international journals and conferences, as well as organizing a research school on the field of parameterized algorithms.